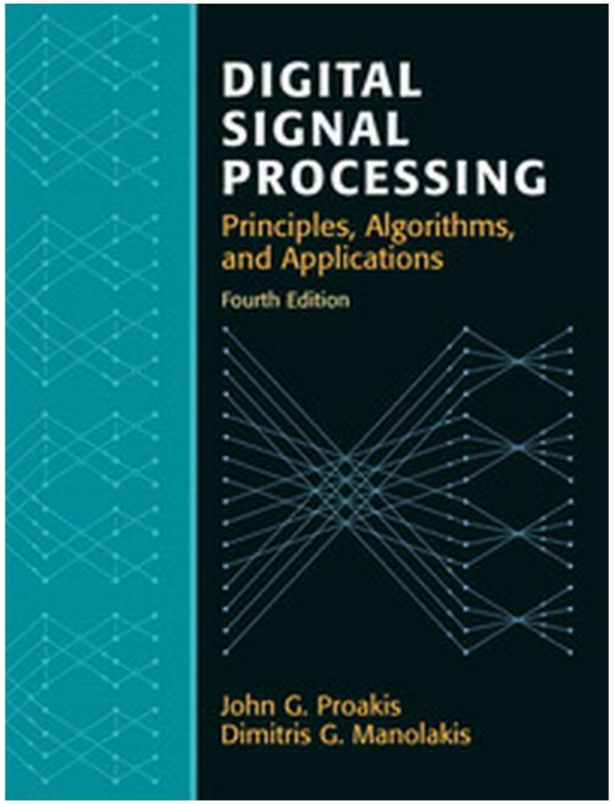
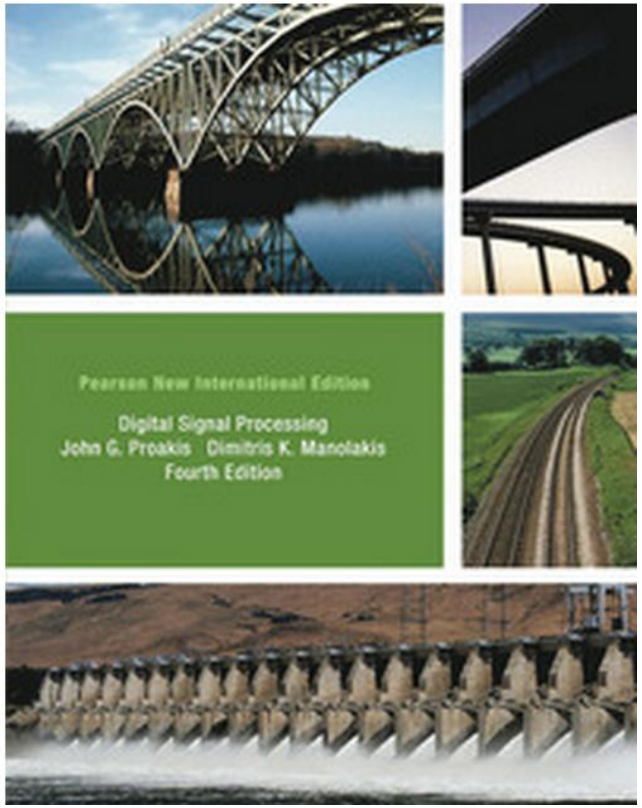
Course Literature

John G. Proakis, Dimitris G. Manolakis, “Digital Signal Processing: Principles, Algorithms, and Applications”, Fourth Edition, Chapters 1–9. Pearson Prentice Hall, ISBN 0-13-187374-1.





Course content

http://www.eit.lth.se/course/eti265

Chapter 1 Introduction.

Chapter 2 Discrete-Time Signals and Systems.

Chapter 3 The z-Transform and its Application to the Analysis of LTI Systems.

Chapter 4 Frequency Analysis of Signals.

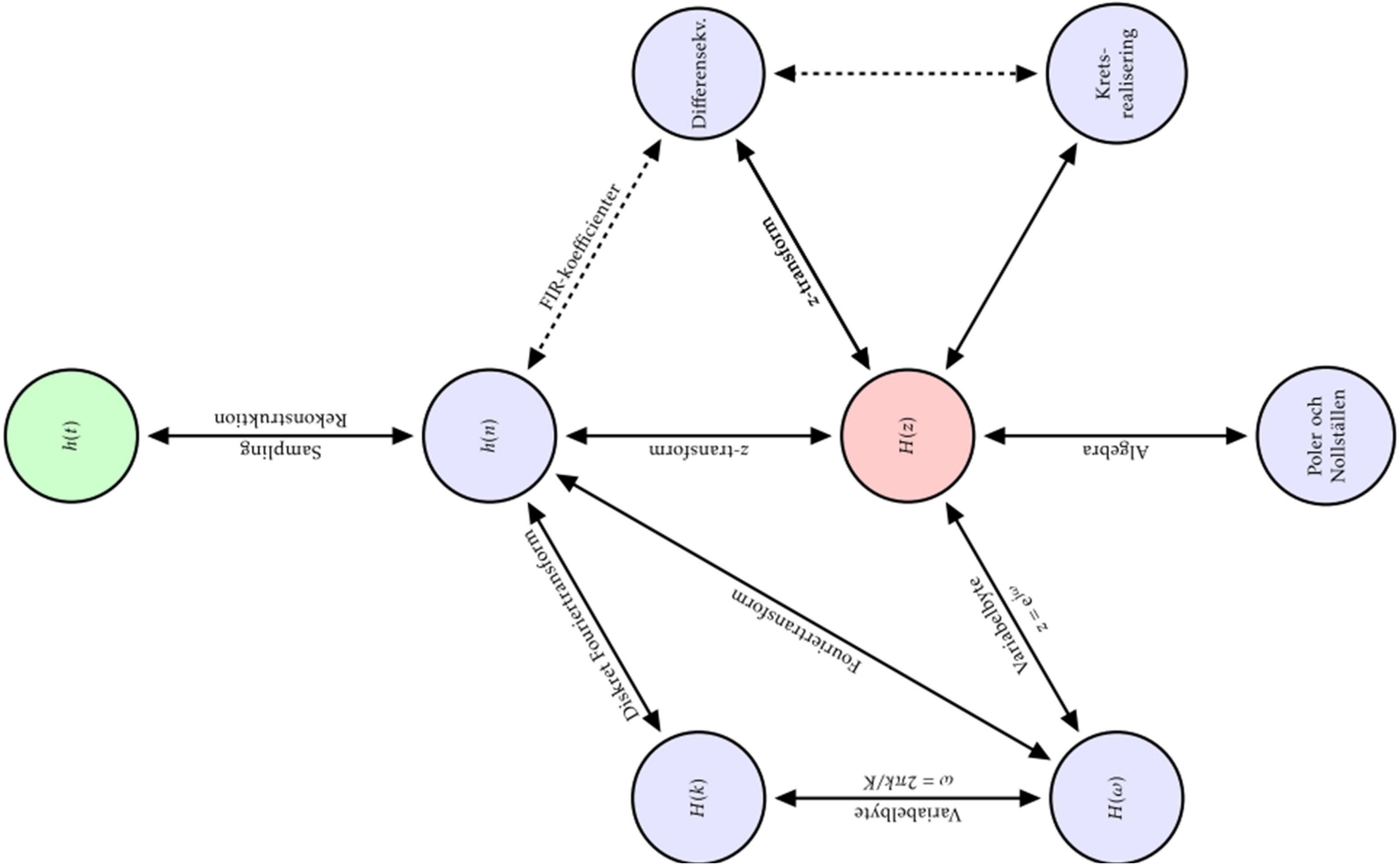
Chapter 5 Frequency-Domain Analysis of LTI Systems.

Chapter 6 Sampling and Reconstruction of Signals.

Chapter 7 The Discrete Fourier transform: Its properties and Applications.

Chapter 8 Efficient Computation of the DFT: Fast Transform Algorithms (not included).

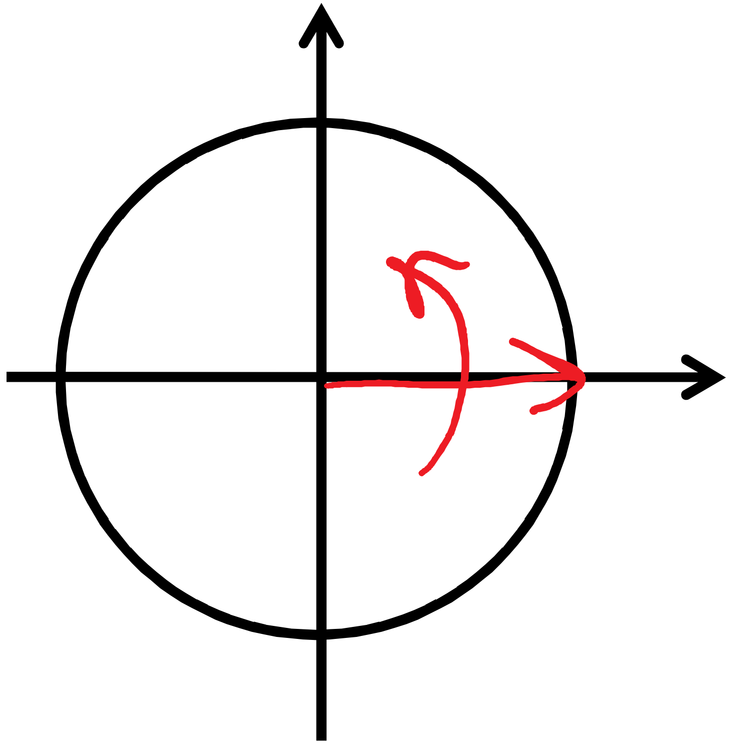
Chapter 9 Implementation of Discrete-Time Systems.



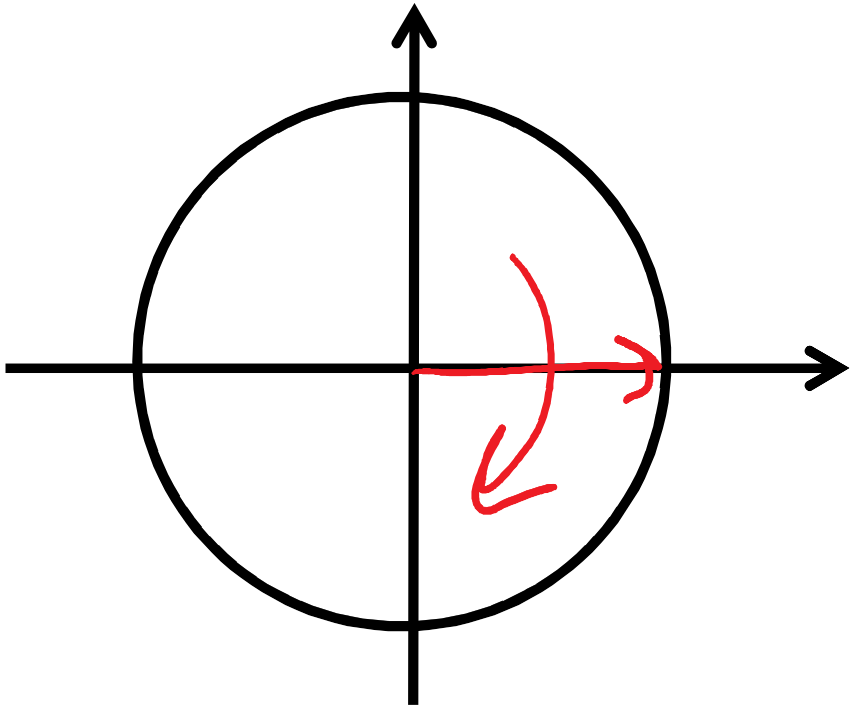
Differensekv.
Krets-
realisering
FIR-koefficienter
z-transform
h(t)
Rekonstruktion
Sampling
h(n)
z-transform
H(z)
Algebra
Poler och
Nollställen
Diskret Fouriertransform
Fouriertransform
Variabelbyte
$z = e^{j\omega}$
H(k)
$\omega = 2\pi k/K$
Variabelbyte
H(ω)

The concept of frequency

Positive frequency



Negative frequency



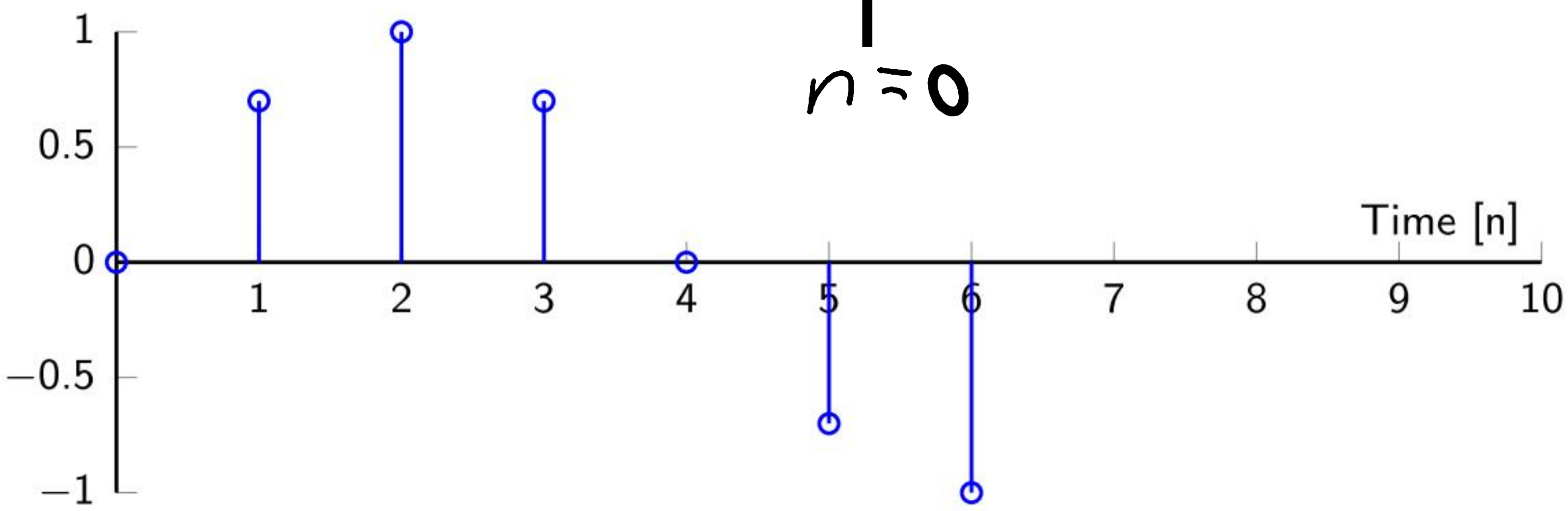
What is a time-discrete signal?

Time-discrete signal

Sine

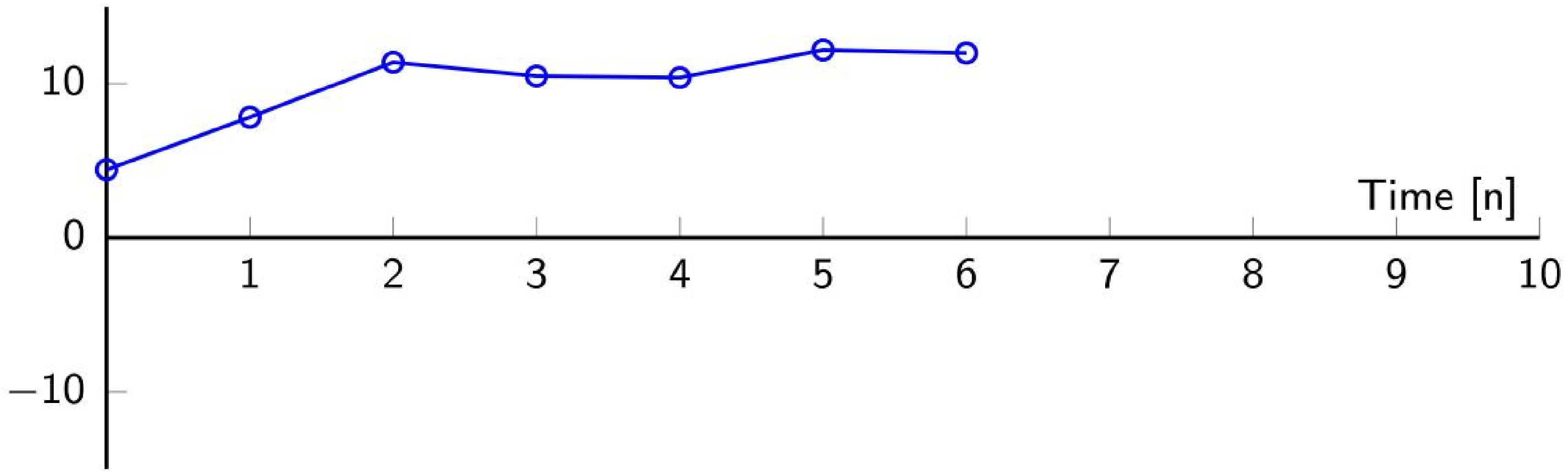
$$x(n) = \sin\left(2\pi \cdot \frac{1}{8} \cdot n\right) \approx \left\{ \; \dots \quad -1 \quad -0.7 \quad \underline{0} \quad 0.7 \quad 1 \quad 0.7 \quad 0 \quad -0.7 \quad \dots \; \right\} \tag{1}$$

n=0 n=1 n=2



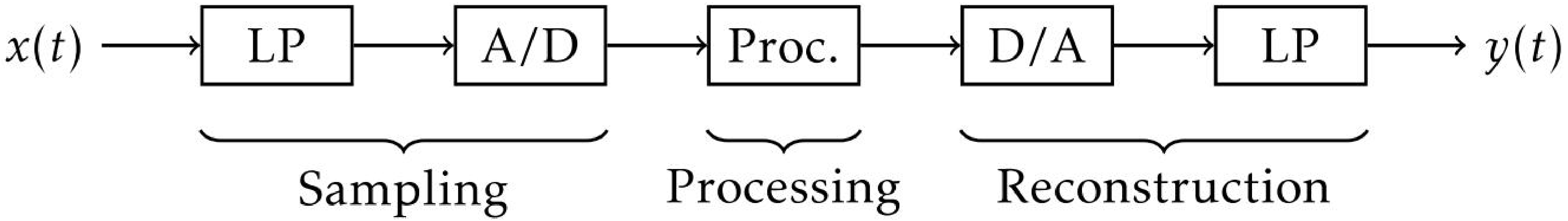
Temperature

$$x(n) = \left\{ \begin{array}{cccccccc} \underline{4.4} & 7.8 & 11.4 & 10.5 & 10.4 & 12.2 & 12.0 & \ldots \end{array} \right\} \tag{2}$$



Time-discrete systems

Digital processing of analog signals.



The digital system.

Example of a systems

$$y(n) = \frac{1}{5} \cdot x(n) + \frac{1}{5} \cdot x(n-1) + \frac{1}{5} \cdot x(n-2) + \frac{1}{5} \cdot x(n-3) + \frac{1}{5} \cdot x(n-4)$$

$$y(n) = \frac{1}{5} \cdot x(n) - \frac{1}{5} \cdot x(n-1) + \frac{1}{5} \cdot x(n-2) - \frac{1}{5} \cdot x(n-3) + \frac{1}{5} \cdot x(n-4)$$

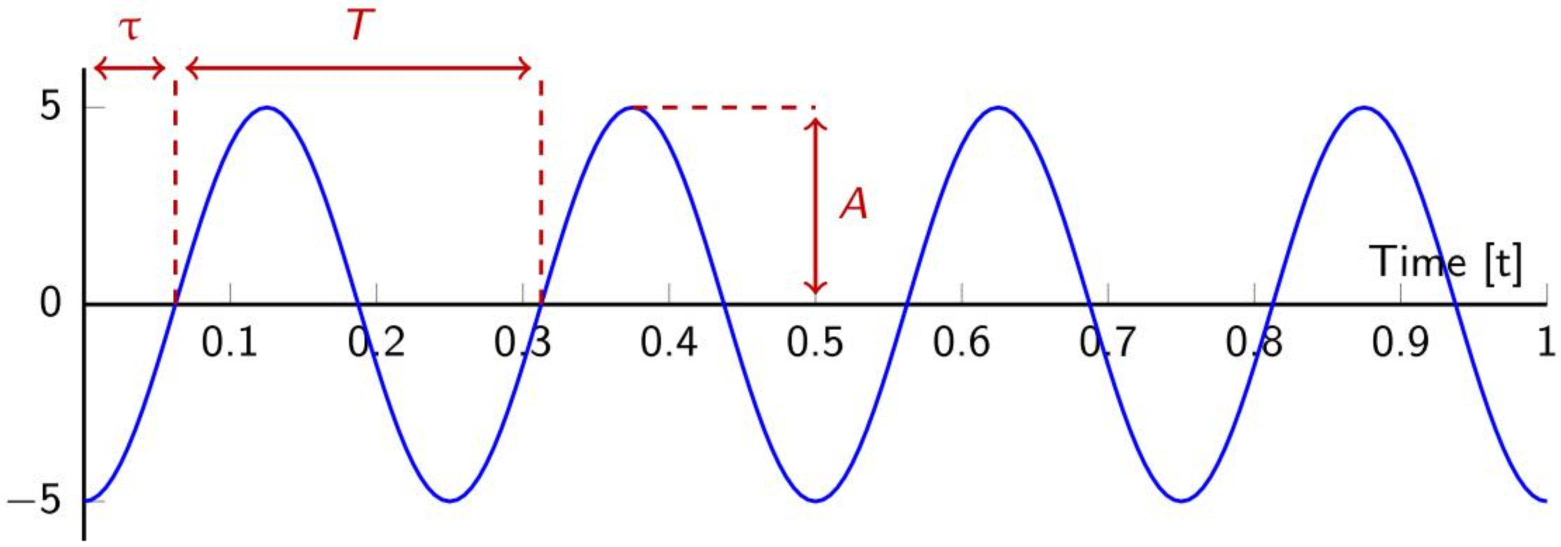
$$y(n) = 0.9y(n-1) + x(n)$$

$$y(n) = 1.1y(n-1) + x(n)$$

Sinusoids

Time signals

$$x(t) = A \cdot \sin(2\pi F t - \Phi) = A \cdot \sin(\Omega t - \Phi) = A \cdot \sin\left(\Omega\left(t - \frac{\Phi}{\Omega}\right)\right) \tag{7}$$

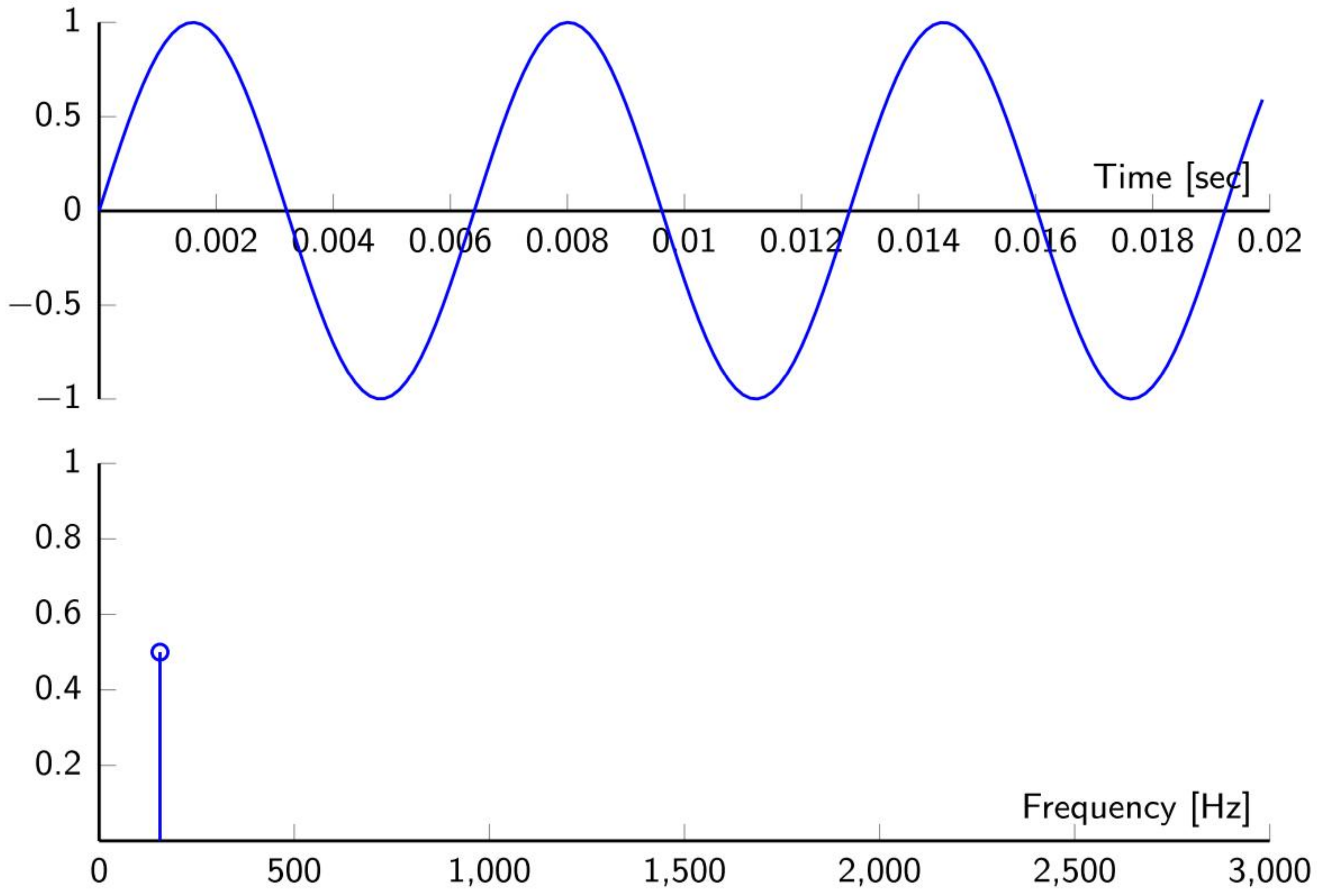


Symbols

A	Amplitude
F	Frequency in Hz
Φ	Phase
$\Omega = 2\pi F$	~~Phase~~ Frequency in rad/s
$T = \frac{1}{F}$	Time period in second
$\tau = \frac{\Phi}{\Omega}$	Time delay in second

Tones

Sine with frequency $F_0 = 156\,\text{Hz}$.

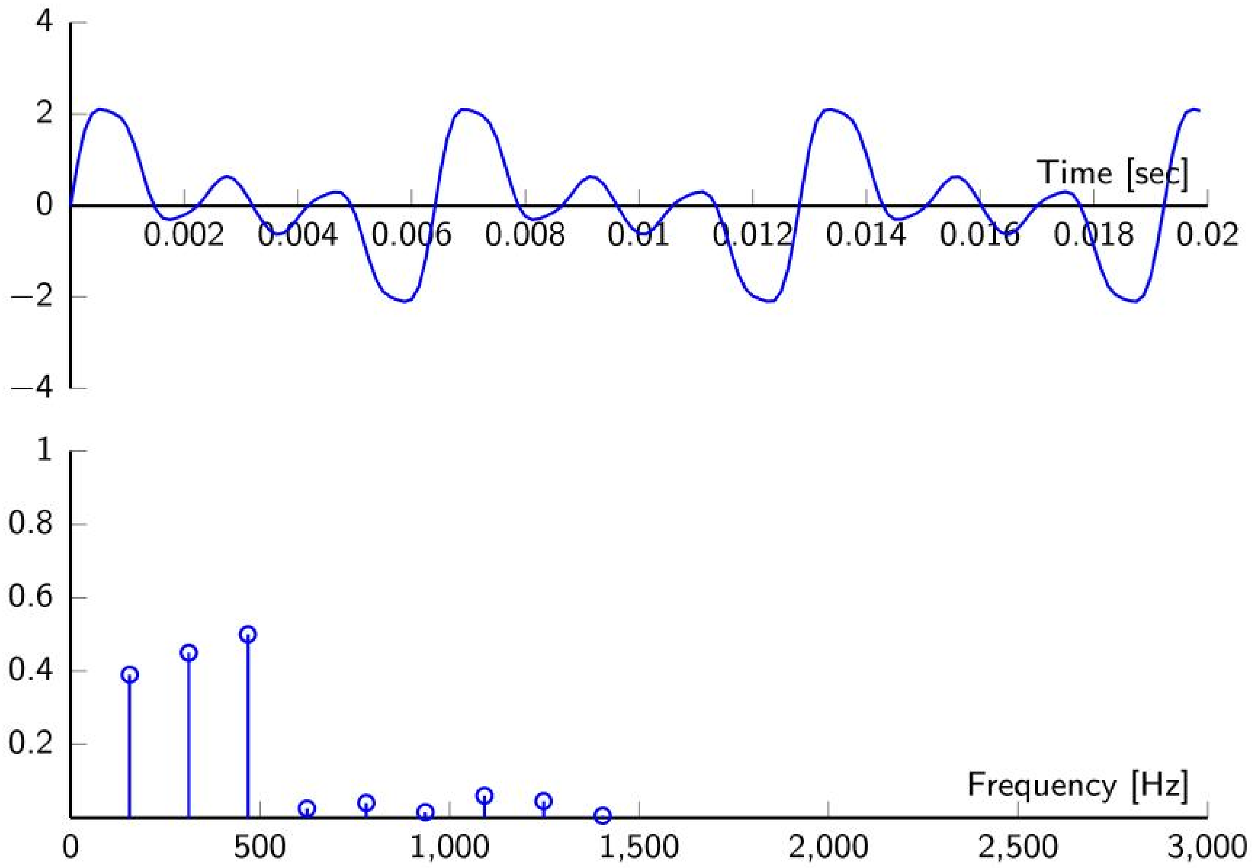


Additive synthesis

Sum of sinusoids of different frequencies, harmonic signal.

$$x(t) = \sum_k a_k \sin(2\pi k F_0 t) \tag{8}$$

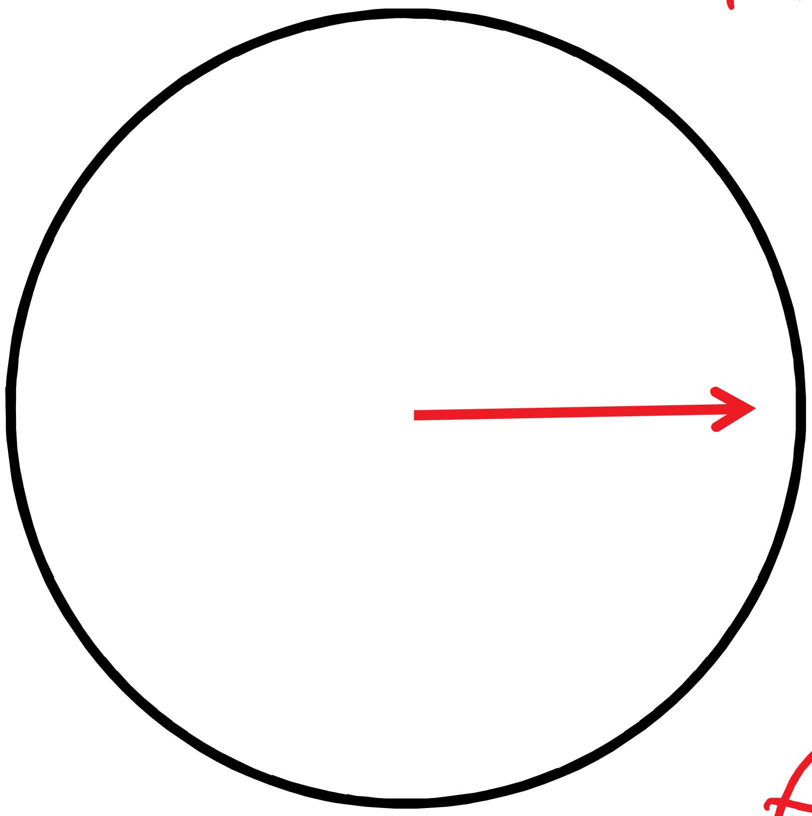
Trombone with fundamental frequency $F_0 = 156\,\text{Hz}$ and 8 harmonics.



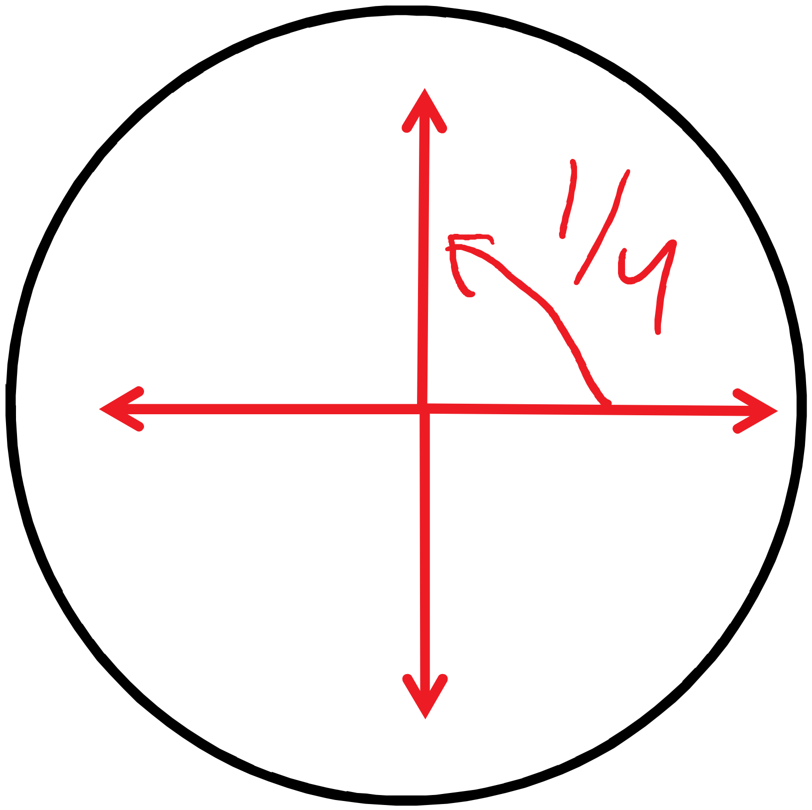
Sampling

4 samples/sec

Real rotation 1 Hz



Sampled rotation



$f = \frac{1}{4}$

Sampling (page 21, 23)

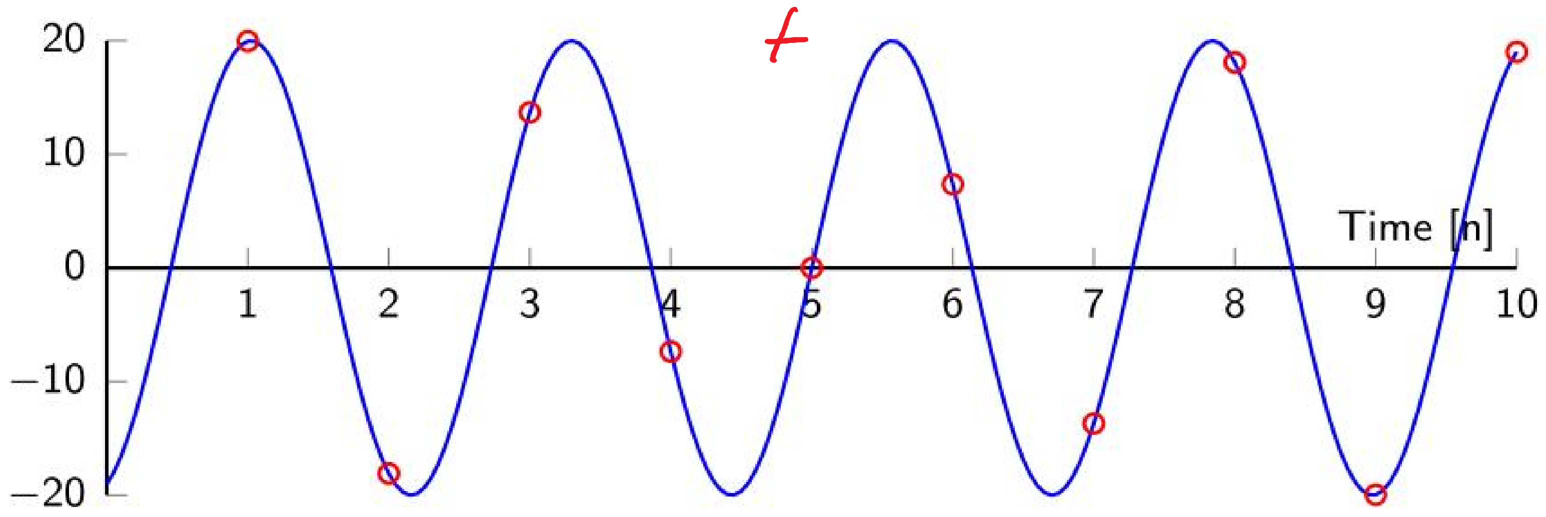
The signal

$$x(t) = 20\cos(2\pi 440t - 0.4\pi) \tag{11}$$

is read with a frequency of $F_s = 1000\,\text{Hz}$, or equivalently with $T_s = \frac{1}{F_s} = \frac{1}{1000} = 0.001\,\text{s}$ between each read.

$$x(n) = x(t \mid t = nT_s = \frac{n}{F_s}) = 20\cos\left(2\pi \cdot \frac{440}{1000} \cdot n - 0.4\pi\right) \tag{12}$$

f = 0,44



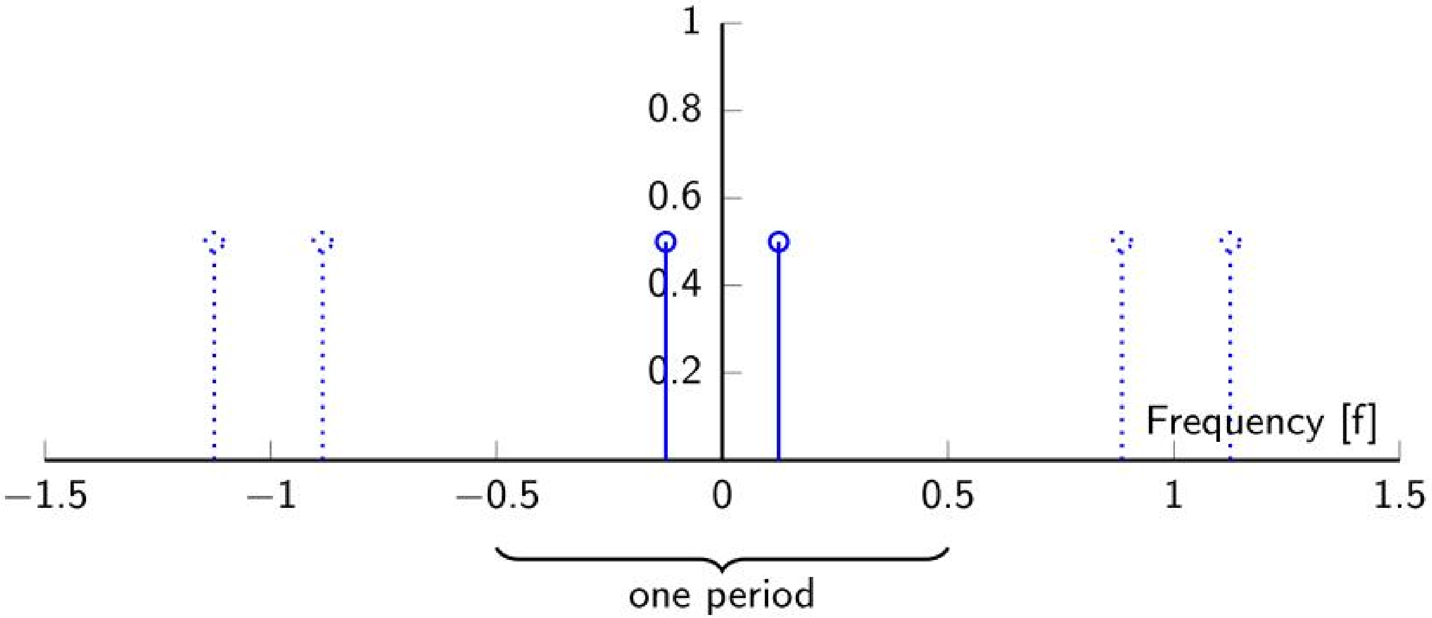
Notations:

$\Omega = 2\pi F$ Frequency and phase for *continuous* signals (real frequency).

$\omega = 2\pi f$ Frequency and phase for *discrete* signaler (digital frequency).

The spectrum for a discrete signal is periodic.

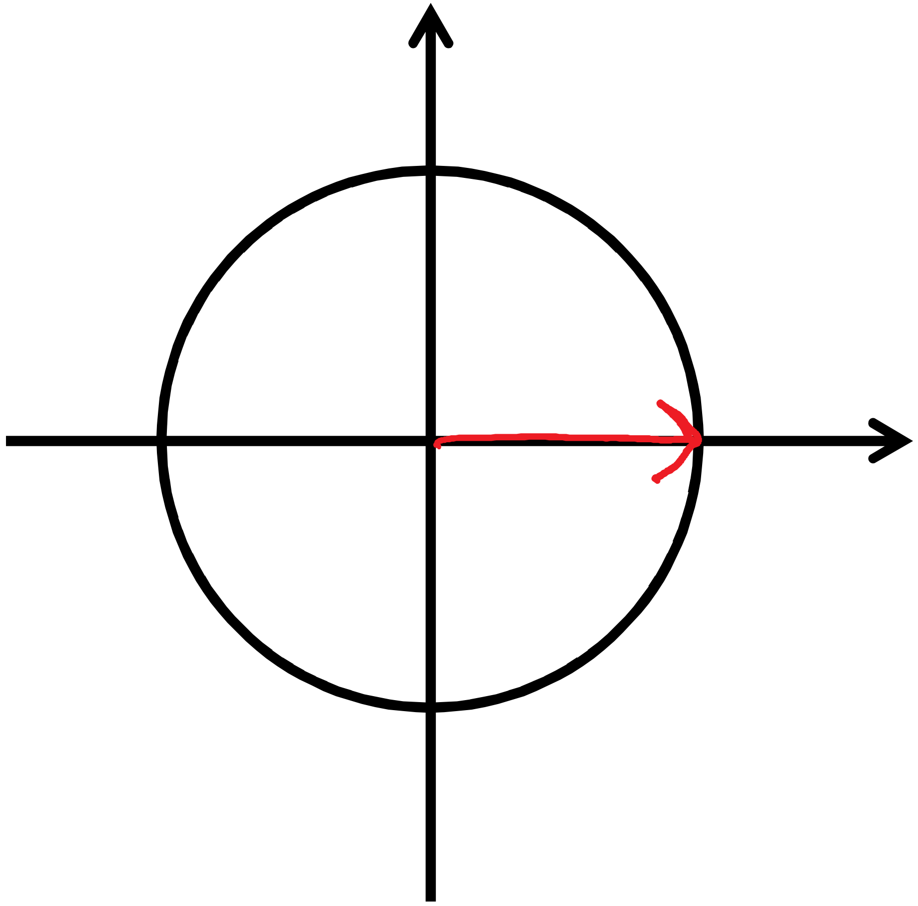
$$x(n) = \cos(2\pi f_0 n) = \cos(2\pi(f_0 + k)n) \qquad \text{for } k \text{ integer} \tag{13}$$



Examples of observed sampled frequencies!

F [Hz]	F_s [Hz]	$\rightarrow$	$f = \frac{F}{F_s}$	$f' \pm k$
1	4	$\rightarrow$	0.25	0.25 ± 0
6.3	1	$\rightarrow$	6.3	$0.3 + 6$
5.8	1	$\rightarrow$	5.8	$-0.2 + 6$
48000	4000	$\rightarrow$	12	$0 + 12$

OBSERVED FREQ



Discrete signals (page 43)

Discrete signals are denoted $x(n)$ (sometimes also $x[n]$).

$$x(n) = \begin{cases} 1 & 0 \leq n < 3 \\ 4 & n = 3 \\ 0 & \text{otherwise} \end{cases} = \{ \ldots \ 0 \ \underline{1} \ 1 \ 1 \ 4 \ 0 \ \ldots \} = \{ \underline{1} \ 1 \ 1 \ 4 \} \tag{14}$$

Impulse

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} = \{ \ldots \ 0 \ \underline{1} \ 0 \ 0 \ 0 \ \ldots \} \tag{15}$$

Step

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} = \{ \ldots \ 0 \ \underline{1} \ 1 \ 1 \ 1 \ \ldots \} \tag{16}$$

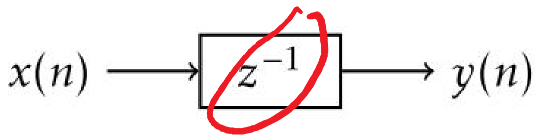
A signal is *causal* if the values are zero for all negative indices.

Using the impulse we can write

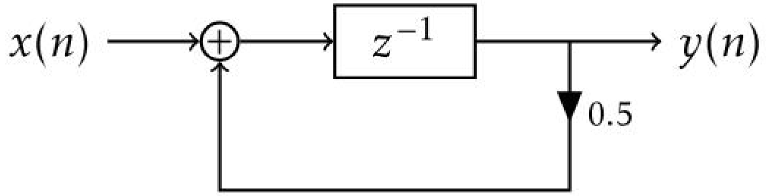
$$x(n) = \{ \underline{1} \quad 4 \quad 1 \} = 1 \cdot \delta(n) + 4 \cdot \delta(n-1) + 1 \cdot \delta(n-2) = \sum_k x(k)\delta(n-k)$$

Example of systems (page 58, 59)

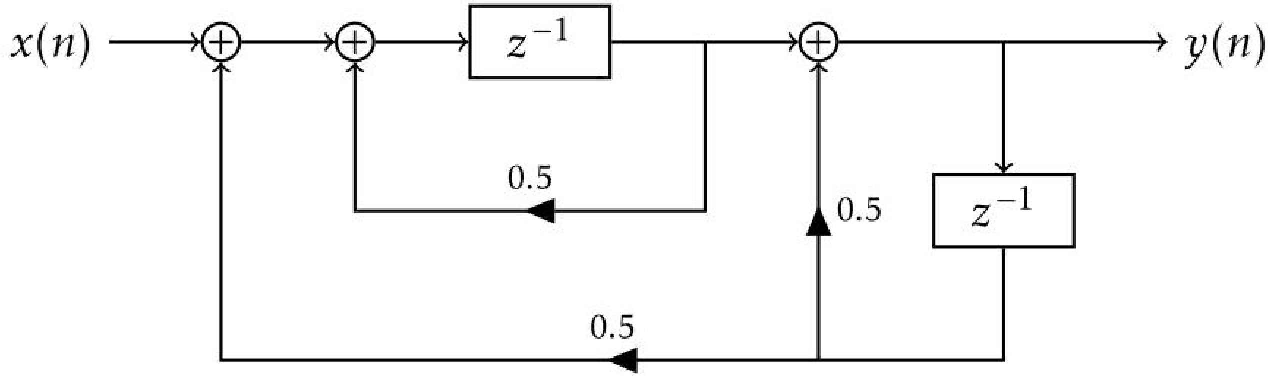
Delay: $y(n) = x(n-1)$



First order system: $y(n) = 0.5 \cdot y(n-1) + x(n-1)$



Second order system



Definitions (page 45)

Energy: A signal is called an *energy signal* if $E < \infty$.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \tag{18}$$

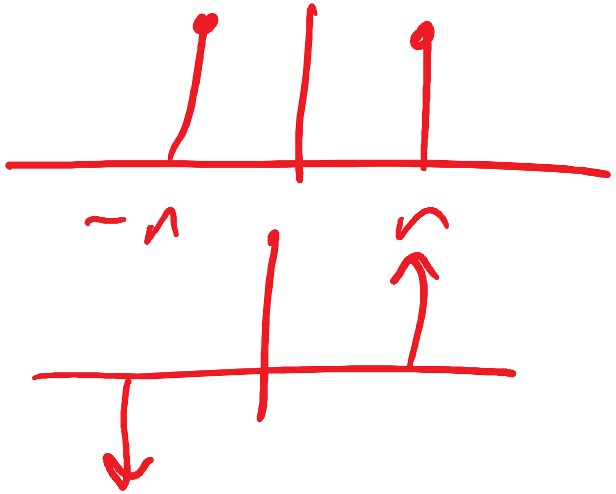
Power: A signal is called a *power signal* if $P < \infty$.

$$P = \frac{1}{N} \cdot \sum_{n=0}^{N} |x(n)|^2 \tag{19}$$

N = one period

Even symmetry

$$x(n) = x(-n) \tag{20}$$



Odd symmetry

$$x(n) = -x(-n) \tag{21}$$

System with finite memory: FIR; for example

Ex. $$y(n) = x(n) + x(n-1)$$

System with infinite memory: IIR; for example

Ex. $$y(n) = 0.5 \cdot y(n-1) + x(n)$$

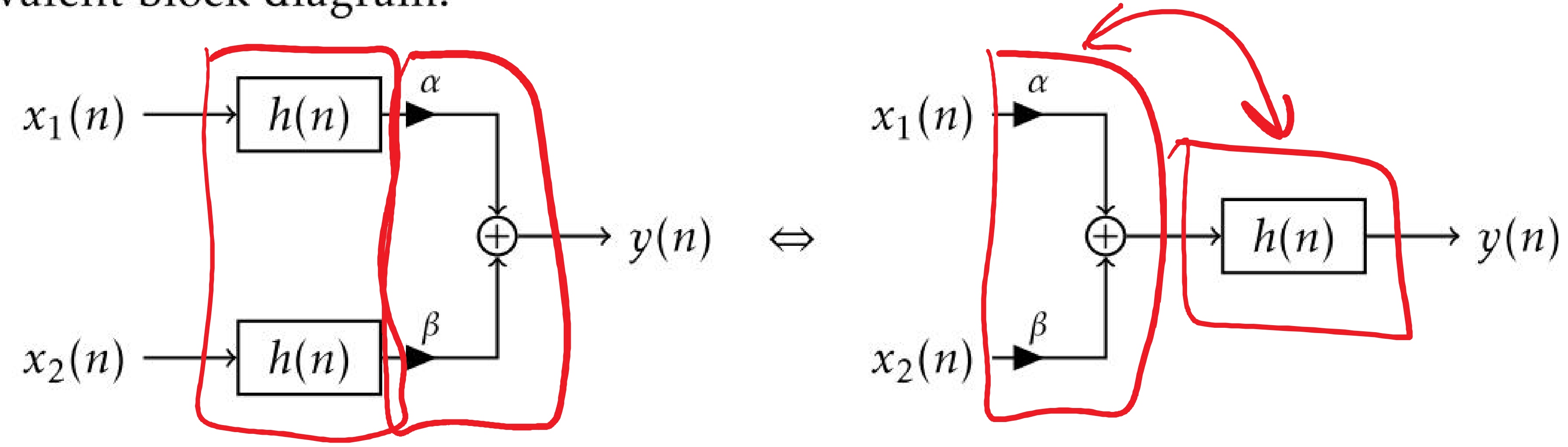
BIBO-stability: Bounded Input Bounded Output.

$$|x(n)| \leq M_x \quad \Leftrightarrow \quad |y(n)| \leq M_y < \infty$$

Linearity

$$x(n) = \alpha x_1(n) + \beta x_2(n) \quad \Leftrightarrow \quad y(n) = \alpha y_1(n) + \beta y_2(n) \tag{24}$$

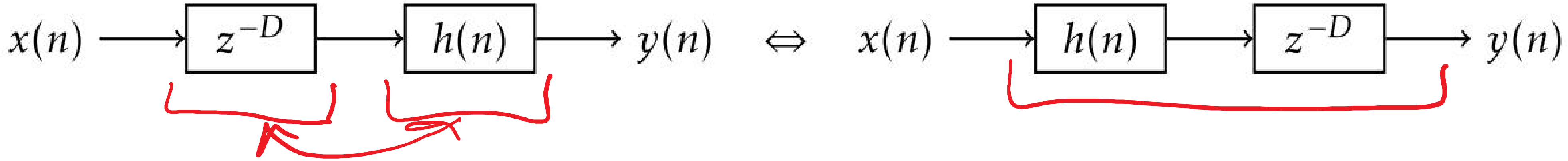
Equivalent block diagram:



Time invariant or Shift invariant

$$x(n) \to x(n-D) \quad \Leftrightarrow \quad y(n) \to y(n-D) \tag{25}$$

Equivalent block diagram:

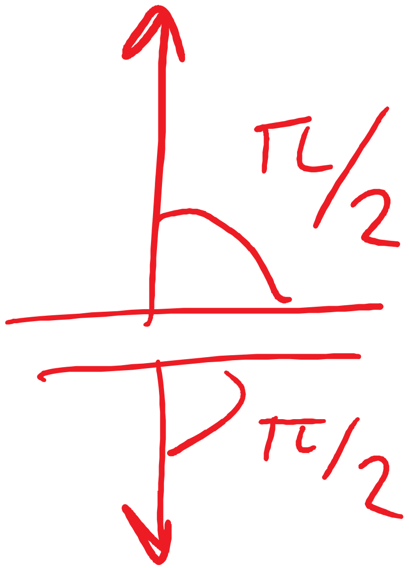


Mathematics in the course

Complex numbers

$$z = a + jb = r \cdot e^{j\Phi} = r \cdot \cos(\Phi) + jr \cdot \sin(\Phi)$$

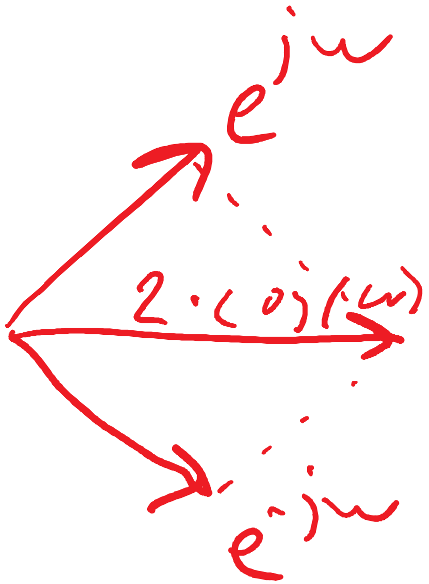
where $r = \sqrt{a^2 + b^2}$ and $\Phi = \arctan(b/a)$ if $a \neq 0$.



Euler's formula

$$\cos(\omega) = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

$$\sin(\omega) = \frac{e^{j\omega} - e^{-j\omega}}{2j}$$



Finite geometric sum

$$S_1 = \sum_{n=0}^{N} a^n = 1 + a + \cdots + a^N = \frac{1 - a^{N+1}}{1 - a} \qquad |a| < 1$$

Proof:

$$S = \sum_{n=0}^{N} a^n = 1 + a + a^2 + \cdots + a^N$$

$$(S - aS) = 1 - a^{N+1}$$

$$a \cdot S = a + a^2 + a^3 + \cdots + a^{N+1}$$

$$S - a \cdot S = 1 - a^{N-1} \qquad \Rightarrow \qquad S = \frac{1 - a^{N+1}}{1 - a}$$

Infinite geometric sum

$$S_2 = \sum_{n=0}^{\infty} a^n = 1 + a + a^2 + \cdots = \frac{1}{1-a} \qquad |a| < 1$$

Proof:

$$\lim_{N\to\infty} \frac{1 - a^{N+1}}{1-a} = \frac{1}{1-a} \qquad \text{if } |a| < 1$$